Algebraic Identities

Unit: Algebra - Student Practice Worksheet

***Overview:** Meera squared 103 in her head in four seconds. Arjun reached for a pen, multiplied 103 by 103 the long way, and got there two minutes later. They got the same answer. Meera hadn't used a calculator, and she hadn't multiplied anything three-digit by three-digit. Arjun wanted to know what she'd actually done.*

Practice Questions (30 Total)

Section A - Easy

Q1. Four tiles for $(a+b)^2$. What is the expanded identity?

$(a+b)^2$ as four tiles

- [ ] **A.** $a^2 + 2ab + b^2$
- [ ] **B.** $a^2 + b^2$
- [ ] **C.** $a^2 - 2ab + b^2$
- [ ] **D.** $a^2 + ab + b^2$

Q2. An identity is true for...

- [ ] **A.** every value of the letters
- [ ] **B.** exactly one value of the letters
- [ ] **C.** only positive values
- [ ] **D.** only whole numbers

Q3. Expand $(a - b)^2$ using the subtraction identity.

- [ ] **A.** $a^2 - b^2$
- [ ] **B.** $a^2 - 2ab + b^2$
- [ ] **C.** $a^2 + 2ab + b^2$
- [ ] **D.** $a^2 - 2ab - b^2$

Q4. In $(a+b)^2$, what coefficient sits on the middle ab tile?

$a^2 \quad + ?\,ab \quad + b^2$

- [ ] **A.** 1
- [ ] **B.** a
- [ ] **C.** b
- [ ] **D.** 2

Q5. Which of these is a true algebraic identity?

- [ ] **A.** $(a + b)^2 = a^2 + b^2$
- [ ] **B.** $a^2 – b^2 = (a – b)^2$
- [ ] **C.** $(a + b)(a – b) = a^2 + b^2$
- [ ] **D.** $(a – b)^2 = a^2 – 2ab + b^2$

Q6. What does (a + b)(a – b) simplify to?

- [ ] **A.** $a^2 + b^2$
- [ ] **B.** $a^2 – 2ab + b^2$
- [ ] **C.** $a^2 – b^2$
- [ ] **D.** 2a

Q7. Meera splits 103 before squaring. Which sum did she use?

$103 \quad \rightarrow \quad 100 + 3$

- [ ] **A.** 50 + 53
- [ ] **B.** 103 + 0
- [ ] **C.** 100 + 3
- [ ] **D.** 90 + 13

Q8. In the identity $(a + b)^2 = a^2 + 2ab + b^2$, squaring b always gives a result that is...

- [ ] **A.** negative
- [ ] **B.** positive or zero
- [ ] **C.** zero
- [ ] **D.** equal to b

Q9. Expand $(x + a)(x + b)$.

- [ ] **A.** $x^2 + ab$
- [ ] **B.** $x^2 + (a + b)x + ab$
- [ ] **C.** $x^2 + (a – b)x + ab$
- [ ] **D.** $x^2 + a^2 + b^2$

Q10. Which identity has no middle term after simplifying, and why do the ab terms vanish?

- [ ] **A.** $(a + b)(a – b)$
- [ ] **B.** $(a + b)^2$
- [ ] **C.** $(a – b)^2$
- [ ] **D.** $(x + a)(x + b)$

Section B - Medium

Q11. Compute 102^2 as $(100+2)^2$ using the identity.

$$102^2 \quad = (100+2)^2$$

- [ ] **A.** 10404
- [ ] **B.** 10204
- [ ] **C.** 10004
- [ ] **D.** 10604

Q12. Arjun computed 97^2 using $a = 100$ and $b = 3$. Which identity did he need?

- [ ] **A.** $(a – b)^2$
- [ ] **B.** $(a + b)^2$
- [ ] **C.** $(a + b)(a – b)$
- [ ] **D.** $(x + a)(x + b)$

Q13. Use an identity to compute 98^2.

- [ ] **A.** 9600
- [ ] **B.** 9604
- [ ] **C.** 9404

- [ ] **D.** 10004

Q14. Expand $(2x+3)^2$ from the a=2x, b=3 tiles.

$(2x+3)^2$

- [ ] **A.** $4x^2 + 6x + 9$
- [ ] **B.** $2x^2 + 12x + 9$
- [ ] **C.** $4x^2 + 9$
- [ ] **D.** $4x^2 + 12x + 9$

Q15. Simplify $(3x + 4y)(3x – 4y)$.

- [ ] **A.** $9x^2 + 16y^2$
- [ ] **B.** $9x^2 – 16y^2$
- [ ] **C.** $3x^2 – 4y^2$
- [ ] **D.** $9x^2 – 24xy – 16y^2$

Q16. Use the difference-of-squares identity to compute 47×53.

- [ ] **A.** 2500
- [ ] **B.** 2509
- [ ] **C.** 2491
- [ ] **D.** 2401

Q17. Expand $(5x–2)^2$ using the subtraction identity.

$(5x–2)^2$

- [ ] **A.** $25x^2 – 20x + 4$
- [ ] **B.** $25x^2 – 10x + 4$
- [ ] **C.** $25x^2 + 20x + 4$
- [ ] **D.** $25x^2 – 4$

Q18. Simplify $(7 – x)(7 + x)$.

- [ ] **A.** $49 + x^2$

- [ ] **B.** $49 - x^2$
- [ ] **C.** $x^2 - 49$
- [ ] **D.** $14 - x^2$

Q19. Expand $(x + 5)(x + 3)$ using the $(x+a)(x+b)$ pattern.

- [ ] **A.** $x^2 + 15x + 8$
- [ ] **B.** $x^2 + 8x + 8$
- [ ] **C.** $x^2 + 8x + 15$
- [ ] **D.** $x^2 + 15x + 15$

Q20. $a+b=10$ and $ab=21$. Find a^2+b^2 from $(a+b)^2 - 2ab$.

- [ ] **A.** 100
- [ ] **B.** 79
- [ ] **C.** 121
- [ ] **D.** 58

Section C - Challenge

Q21. $a^2+b^2=41$ and $ab=20$. Find $(a+b)^2$.

$a^2+b^2=41$ $ab=20$

- [ ] **A.** 81
- [ ] **B.** 41
- [ ] **C.** 61
- [ ] **D.** 121

Q22. Which expression equals $(a - b)^2 + 4ab$?

- [ ] **A.** $(a - b)^2$
- [ ] **B.** $a^2 - b^2$
- [ ] **C.** $(a + b)^2$
- [ ] **D.** $2a^2 + 2b^2$

Q23. If $a - b = 4$ and $a^2 + b^2 = 40$, find ab.

- [ ] **A.** 8

- [ ] **B.** 12
- [ ] **C.** 16
- [ ] **D.** 20

Q24. Simplify $(a+b)^2 - (a-b)^2$ using the two expansions.

$$(a+b)^2 - (a-b)^2$$

- [ ] **A.** $2a^2$
- [ ] **B.** $2b^2$
- [ ] **C.** 0
- [ ] **D.** $4ab$

Q25. Meera wants 205 × 195 without long multiplication. Which pair (a, b) lets her use the difference-of-squares identity directly?

- [ ] **A.** a = 205, b = 195
- [ ] **B.** a = 100, b = 95
- [ ] **C.** a = 400, b = 5
- [ ] **D.** a = 200, b = 5

Q26. Use identities to compute 999^2 without long multiplication.

- [ ] **A.** 999998
- [ ] **B.** 998999
- [ ] **C.** 998001
- [ ] **D.** 997001

Q27. Square (x+7) minus square (x–7). Remaining area?

big square minus small square

- [ ] **A.** $28x$
- [ ] **B.** $x^2 + 49$
- [ ] **C.** $(x+7)(x-7) = x^2 - 49$
- [ ] **D.** $x^2 - 14x + 49$

Q28. Simplify $(x + y)^2 – 2xy$.

- ☐ **A.** $x^2 + y^2$
- ☐ **B.** $x^2 – y^2$
- ☐ **C.** $(x – y)^2$
- ☐ **D.** $x^2 + 2xy + y^2$

Q29. If $a + b = 9$ and $a – b = 3$, use identities to find $a^2 – b^2$ directly (without first finding a and b).

- ☐ **A.** 12
- ☐ **B.** 27
- ☐ **C.** 6
- ☐ **D.** 81

Q30. If $x + 1/x = 5$, square both sides to find $x^2 + 1/x^2$, subtracting the 2.

- ☐ **A.** 25
- ☐ **B.** 23
- ☐ **C.** 27
- ☐ **D.** 10

Answer Key

Q1: (A) (a) $a^2 + 2ab + b^2$. Forgetting the 2ab cross term (option b) is the common mistake.

Q2: (A) (a) every value of the letters. That's what separates an identity from an equation you solve for a specific answer.

Q3: (B) (b) $a^2 – 2ab + b^2$. Note b^2 stays positive — squaring $–b$ always gives a positive result, which trips up option (d).

Q4: (D) (d) 2. The middle term is 2ab, not ab — forgetting this 2 is the single most common identity mistake.

Q5: (D) (d) $(a – b)^2 = a^2 – 2ab + b^2$ is the genuine identity. The others drop or misplace the cross term.

Q6: (C) (c) $a^2 – b^2$. The middle terms $+ab$ and $–ab$ cancel out, leaving no ab term at all — that's the whole point of this identity.

Q7: (C) (c) 100 + 3. She chose a round number (100) plus a small remainder, which makes both a^2 and 2ab easy to compute in your head.

Q8: (B) (b) positive or zero. Any real number squared cannot be negative, which is exactly why b^2 stays positive even in $(a – b)^2$.

Q9: (B) (b) $x^2 + (a + b)x + ab$. The middle coefficient is the SUM of a and b, and the last term is their product.

Q10: (A) (a) $(a + b)(a – b)$. Its two middle terms cancel, giving purely $a^2 – b^2$ with nothing in between.

Q11: (A) (a) 10404. $(100+2)^2 = 10000 + 400 + 4 = 10404$. Option (b) is what you get if you forget the middle 2ab term entirely.

Q12: (A) (a) $(a – b)^2$. Since $97 = 100 – 3$, squaring it needs the subtraction identity, giving $10000 – 600 + 9 = 9409$.

Q13: (B) (b) 9604. $(100–2)^2 = 10000 – 400 + 4 = 9604$. Option (a) drops the +4 (the b^2 term), a common shortcut error.

Q14: (D) (d) $4x^2 + 12x + 9$. Here $a = 2x$ and $b = 3$, so $a^2 = 4x^2$, $2ab = 12x$, $b^2 = 9$. Option (a) forgets to double 2x when squaring it.

Q15: (B) (b) $9x^2 – 16y^2$. This is a – b squared pattern: $(3x)^2 – (4y)^2 = 9x^2 – 16y^2$, with no middle term.

Q16: (C) (c) 2491. Write 47×53 as $(50–3)(50+3) = 50^2 – 3^2 = 2500 – 9 = 2491$. Option (a) forgets to subtract 9 at all.

Q17: (A) (a) $25x^2 – 20x + 4$. Here $a = 5x$, $b = 2$: $a^2 = 25x^2$, $2ab = 20x$, $b^2 = 4$, all with a minus in the middle.

Q18: (B) (b) $49 – x^2$. Written as $(a–b)(a+b)$ with $a=7$, $b=x$, this is $a^2 – b^2 = 49 – x^2$, not the reverse order in option (c).

Q19: (C) (c) $x^2 + 8x + 15$. The middle coefficient is the sum $5+3=8$, and the constant is the product $5\times3=15$ — option (a) has swapped them.

Q20: (D) (d) 58. Since $(a+b)^2 = a^2 + 2ab + b^2$, rearrange to $a^2 + b^2 = (a+b)^2 – 2ab = 100 – 42 = 58$.

Q21: (A) (a) 81. $(a+b)^2 = a^2 + 2ab + b^2 = 41 + 2(20) = 41 + 40 = 81$. Option (c) mistakenly adds ab once instead of doubling it.

Q22: (C) (c) $(a + b)^2$. Expand: $a^2 – 2ab + b^2 + 4ab = a^2 + 2ab + b^2$, which is exactly $(a+b)^2$.

Q23: (B) (b) 12. $(a–b)^2 = a^2 – 2ab + b^2$, so $16 = 40 – 2ab$, giving $2ab = 24$ and $ab = 12$. Option (a) forgets to divide by 2 at the end.

Q24: (D) (d) 4ab. $(a^2+2ab+b^2) – (a^2–2ab+b^2) = 4ab$; the a^2 and b^2 terms cancel, leaving only the doubled middle terms.

Q25: (D) (d) a = 200, b = 5, since 205 = 200+5 and 195 = 200–5, giving $(200+5)(200–5) = 200^2 – 5^2 = 40000 – 25 = 39975$.

Q26: (C) (c) 998001. $(1000–1)^2 = 1000000 – 2000 + 1 = 998001$. Option (d) makes an arithmetic slip subtracting 2000 twice instead of once.

Q27: (A) (a) 28x. Remaining area = $(x+7)^2 – (x–7)^2 = 4·x·7 = 28x$, not the product of the two sides.

Q28: (A) (a) $x^2 + y^2$. Expanding gives $x^2 + 2xy + y^2 – 2xy$, and the 2xy terms cancel, leaving $x^2 + y^2$, not the full $(x–y)^2$ of option (b).

Q29: (B) (b) 27. $a^2 – b^2 = (a+b)(a–b) = 9 × 3 = 27$ — no need to solve for a and b separately first, which is the whole speed advantage of the identity.

Q30: (B) (b) 23. Square both sides: $(x + 1/x)^2 = x^2 + 2 + 1/x^2 = 25$, so $x^2 + 1/x^2 = 25 – 2 = 23$. Option (a) forgets to subtract the 2.

You've got this! Keep learning and shining! - Ruchi Math Coaching